

Question Paper

Quantitative Methods – I (131) : April 2003

Section A : Basic Concepts (40 Points)

- This section consists of questions with serial number 1 - 40.
- Answer all questions.
- Each question carries one point.

1. The set of natural numbers
 - a. Includes negative numbers
 - b. Includes zero
 - c. Includes all the numbers starting from 1 and obtained by adding 1 to the previous number
 - d. Includes only odd numbers
 - e. Includes only even numbers.
2. The set of real numbers
 - a. Excludes the natural numbers
 - b. Excludes the integers
 - c. Excludes the rational numbers
 - d. Excludes the whole numbers
 - e. Excludes the complex numbers.
3. $(a + b) + c = a + (b + c)$ is known as the
 - a. Associative property of addition
 - b. Commutative property of addition
 - c. Distributive property
 - d. Identity property of addition
 - e. Inverse property of addition.
4. Which property states that for every number **a**, **a** $\neq$ 0, there exists another number $\frac{1}{a}$ such that the multiplication of **a** and $\frac{1}{a}$ is equal to 1?
 - a. Commutative property of multiplication
 - b. Associative property of multiplication
 - c. Identity property of multiplication
 - d. Inverse property of multiplication
 - e. Cancellation property of multiplication.
5. Which of the following intervals, defined by the end points **a** and **b**, excludes the real numbers which are less than or equal to **a**, or, greater than or equal to **b**?
 - a. (a, b)
 - b. [a, b]
 - c. (a, b]
 - d. [a, b)
 - e. None of the above.
6. The smallest number that can be divided by each of a group of numbers without leaving a remainder is called
 - a. A factor of the quantities
 - b. The highest common factor of the quantities
 - c. The least common multiple of the quantities
 - d. The average of the quantities
 - e. The sum of all the quantities.

7. In which of the following conditions will a quadratic equation of the form, $ax^2 + bx + c = 0$, have a single root?
- $b = ac$
 - $b^2 = ac$
 - $\frac{b^2}{4} = ac$
 - $\frac{b}{4} = ac$
 - $\frac{b}{4} = a$
8. A series of numbers is said to be in arithmetic progression if
- The consecutive numbers increase as multiples of a fixed number
 - The consecutive numbers decrease as factors of a fixed number
 - The consecutive numbers assume positive and negative signs alternately
 - The consecutive numbers are multiples of 100
 - The consecutive numbers in the series differ by a constant value.
9. In an arithmetic progression any number can be expressed as
- A multiple of the first number of the progression
 - A factor of the first number of the progression
 - A sum of the first number and a multiple of the common difference
 - A multiple of a constant quantity, not necessarily the first number of the progression
 - A factor of a constant quantity, not necessarily the first number of the progression.
10. The first term of an arithmetic progression (A.P.) is **a**, the common difference is **d** and there are **n** terms in it. If the sum of the terms of the A.P. is zero then which of the following is **true**?
- $a = -n$
 - $a = 2n$
 - $a = -d$
 - $a = nd$
 - $a = -\frac{(n-1)}{2}d$
11. If three quantities, **a**, **b** and **c**, are in geometric progression then
- $a = b + c$
 - $a = b \cdot c$
 - $a \cdot b = c$
 - $b^2 = a \cdot c$
 - $2b = a + c$
12. The reciprocals of the terms in any harmonic progression are
- Odd numbers in multiples of 3
 - Even numbers in multiples of 4
 - Prime numbers
 - In geometric progression
 - In arithmetic progression.
13. If three quantities **a**, **b** and **c**, are in harmonic progression then
- $a - b = b - c$
 - $a(b - c) = c(a - b)$
 - $ac = a + b$
 - $bc = a + c$
 - $ab = b + c$

14. If A, G and H are the arithmetic mean, geometric mean and harmonic mean respectively, between two given quantities, **a** and **b**, then
- $G^2 = A - H$
 - $G^2 = H - A$
 - $G^2 = \frac{A}{H}$
 - $G^2 = \frac{H}{A}$
 - $G^2 = A \cdot H$
15. n_{C_r} is equal to
- $n_{P_r} \times r$
 - $n_{P_r} \times r!$
 - $n_{P_r} \times n$
 - $n_{P_r} \div r!$
 - $n_{P_r} \times n!$
16. If $a = \log_b c$ then
- $a = b \times c$
 - $c = a \times b$
 - $b = a \times c$
 - $c = b^a$
 - $c = a^b$
17. $\log_a(M/N)$ is equal to
- $\log_a M + \log_a N$
 - $\log_a M - \log_a N$
 - $\log_a M \div \log_a N$
 - $\log_a N - \log_a M$
 - $\log_a N \div \log_a M$
18. $\log_a 1$ is equal to
- 0
 - 1
 - a
 - A negative number
 - 10
19. The absolute value of a number is
- A multiple of the number
 - A factor of the number
 - The product of the number with an integer
 - The magnitude of the number without its sign
 - The sum of the number with another number.

20. For a real number, a , the interval $[a, +\infty)$ indicates
- Set of all real numbers less than a
 - Set of all real numbers less than or equal to a
 - Set of all even numbers greater than a
 - Set of all odd numbers greater than a
 - Set of all real number greater than or equal to a .
21. The range of a function, $y = f(x)$, includes
- The values which may be assumed by x
 - The values of y which can be associated with the values of x
 - The set of positive integers only
 - The set of negative integers only
 - The set of rational numbers only.
22. For a function, $y = f(x)$, every member in the domain is associated with
- Only one member in the range
 - Only two members in the range
 - Only three members in the range
 - Only four members in the range
 - Only five members in the range.
23. The limit of a function, $y = f(x)$, at a point $x = a$, is the
- Minimum value of the function for all values of x greater than a
 - Minimum value of the function for all values of x less than a
 - Maximum value of the function for all values of x greater than a
 - Maximum value of the function for all values of x less than a
 - Value which the function approaches if x approaches a .
24. The derivative of a function, $y = f(x)$, at a point $x = a$ is equal to
- The limit of the function at $x = a$
 - The value of the function at $x = a$
 - $\lim_{x \rightarrow a} \frac{f(x)}{a}$
 - $\lim_{x \rightarrow a} \frac{f(x)}{f(a)}$
 - $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$
25. For a function, $y = f(x)$, $f'(x) > 0$ and $f''(x) < 0$. It can be said that $f(x)$ is
- Constant for all values of x
 - Decreasing at an increasing rate
 - Decreasing at a decreasing rate
 - Increasing at an increasing rate
 - Increasing at a decreasing rate.
26. A function, $y = f(x)$, is said to have a relative maxima or minima at $x = a$, if
- $f'(x) = 0$ at $x = a$
 - $f'(x) > 0$ at $x = a$
 - $f'(x) < 0$ at $x = a$
 - $f(x) = 0$ at $x = a$
 - $f(x) < 0$ at $x = a$.

27. Which of the following is **not** an assumption underlying linear programming?
- The objective function can be expressed in terms of a linear equation
 - The constraints can be expressed in terms of linear equations or inequations
 - The usage of the resources is known with certainty
 - The decision variables can take only integral values
 - The total usage of a resource is the sum of the resources used by each decision variable.
28. Which of the following is **false** with regard to the graphical method of solving linear programming problems?
- The graphical method can be used when there are two decision variables
 - The decision variables are represented by the horizontal and vertical axes
 - The points lying outside the feasible region satisfy all the constraints
 - The feasible region lies in the first quadrant of the graphical plane
 - None of the above.
29. The optimal solution, according to the graphical solution to a linear programming problem,
- Occurs at the corner points of the feasible region
 - Occurs near the centre of the feasible region
 - Occurs outside the feasible region
 - Occurs at the horizontal axis
 - Occurs at the vertical axis.
30. Which of the following is **false** with regard to the simplex method of solving linear programming problems on profit maximization?
- The constraint inequations are converted into equations by using slack variables
 - It is applicable to problems involving two or more decision variables
 - The variable to enter solution is indicated by the values in the solution column
 - The profit contribution of the slack variables is zero
 - None of the above.
31. For a distribution of data, if the arithmetic mean $>$ median $>$ mode, then which of the following is **true**?
- The distribution is symmetrical
 - The distribution is positively skewed
 - The distribution is negatively skewed
 - Most of the values are concentrated at the higher end of the distribution
 - None of the above.
32. Which of the following represents a frequency distribution, using rectangles?
- Scatter diagram
 - Frequency polygon
 - Ogive
 - Histogram
 - None of the above.
33. If every item in a data set is multiplied by a constant, K, then, the mean of the resulting data set will be
- $K + \text{Mean of the original data set}$
 - $K - \text{Mean of the original data set}$
 - $K \times \text{Mean of the original data set}$
 - $K \div \text{Mean of the original data set}$
 - $\text{Mean of the original data set} \div K$.

34. Which of the following measures of central tendency represents the most frequently occurring value in the data set?
- Arithmetic mean
 - Geometric mean
 - Harmonic mean
 - Median
 - Mode.
35. Which of the following measures of central tendency is appropriate for computing the average rate of change of quantities which vary over time?
- Arithmetic mean
 - Geometric mean
 - Harmonic mean
 - Median
 - Mode.
36. The appropriate mean for a set of ratios which have been calculated with the same denominators is
- Simple arithmetic mean
 - Geometric mean
 - Simple harmonic mean
 - Median
 - Mode.
37. The Bienayme-Chebyshev theorem is not applicable when the difference about the mean is
- 1.0 standard deviation
 - 1.5 standard deviations
 - 2.0 standard deviations
 - 2.5 standard deviations
 - 3.0 standard deviations.
38. Which of the following is calculated with the help of Baye's theorem?
- Central tendency
 - Dispersion
 - Posterior probability
 - Prior probability
 - None of the above.
39. Which of the following enables us to compare the variability of two or more sets of data?
- Variance
 - Standard deviation
 - Coefficient of variation
 - Mean absolute deviation
 - None of the above.
40. In which of the following conditions two events, A and B, are said to be mutually exclusive?
- $P(A) + P(B) > 1.00$
 - $P(A) = P(B)$
 - $P(A/B) = 0$
 - $P(B/A) = 0$
 - Both (c) and (d) above.

END OF PART A

Section B : Problems (60 Points)

- This section consists of questions with serial number 41 - 82.
- Answer all questions.
- Points are indicated against each question.

41. The arithmetic mean and geometric mean of two values are 4 and 6 respectively. What is the harmonic mean?

- a. $\frac{2}{3}$
- b. $\frac{3}{2}$
- c. 5
- d. 9
- e. 10

(1 point)

42. $\log_2 9 \times \log_3 2 =$

- a. 1
- b. 2
- c. 3
- d. 4
- e. 5

(1 point)

43. The second and fourth terms of a geometric progression are 10 and 40 respectively. What is the fifth term?

- a. 12
- b. 15
- c. 20
- d. 24
- e. 80

(1 point)

44. What are the possible number of permutations of 5 different things taking 2 things at a time?

- a. 10
- b. 20
- c. 30
- d. 40
- e. 60

(1 point)

45. If $f(x) = e^{x^2+5}$ then $f'(x) =$

- a. $2x \cdot e^{x^2+5}$
- b. $2x$
- c. e^{x^2}
- d. e^{x^2+5}
- e. 2

(1 point)

46. The variance of a set of data is 9 and the arithmetic mean is 5. What is the coefficient of variation?

a. 6%
b. 60%
c. 80%
d. 150%
e. 167%

(1 point)

47. The probabilities of two non-mutually exclusive and independent events, A and B, are 0.2 and 0.3 respectively. What is the probability of their simultaneous occurrence?

a. 0.01
b. 0.05
c. 0.06
d. 0.15
e. 0.08

(1 point)

48. The variance of a set of data is 3. If every item is doubled then, what will be the variance of the resulting data set?

a. $\frac{3}{4}$
b. $\frac{3}{2}$
c. 3
d. 6
e. 12

(1 point)

49. $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x - 3} =$

a. 1
b. 2
c. 3
d. 4
e. 5

(1 point)

50. A and B are two non-mutually exclusive and independent events.

$$P(A \text{ or } B) = 0.80$$

$$P(A) = 0.50$$

$$P(A \text{ and } B) = 0.30$$

What is the value of P(B)?

a. 0.50
b. 0.60
c. 0.70
d. 0.80
e. 0.24

(1 point)

Questions 51 – 57 are based on the following information:

The scientists at Bharat Biotech Ltd. have developed three new drugs, X, Y and Z, which reduce the probability of heart attack. Drug X reduces the probability of heart attack by 20 percent, Drug Y reduces the probability of heart attack by 25 percent and Drug Z reduces the probability of heart attack by 35 percent. A random sample of Indian patients is selected such that each patient has a 75 percent probability of heart attack. The patients are divided into three groups in the ratio of 30:30:40, and these groups are prescribed the drugs X, Y and Z respectively.

51. What is the probability that a randomly selected patient has been prescribed Drug X and the patient will have a heart attack?
- a. 0.165
 - b. 0.184
 - c. 0.250
 - d. 0.305
 - e. 0.550.

(1 point)

52. What is the probability that a randomly selected patient has been prescribed Drug Y and the patient will have a heart attack?
- a. 0.150
 - b. 0.165
 - c. 0.256
 - d. 0.308
 - e. 0.505.

(1 point)

53. What is the probability that a randomly selected patient has been prescribed Drug Z and the patient will have a heart attack?
- a. 0.150
 - b. 0.160
 - c. 0.305
 - d. 0.402
 - e. 0.506

(1 point)

54. What is the probability that a randomly selected patient, who has been prescribed either of the three drugs, will have a heart attack?
- a. 0.150
 - b. 0.165
 - c. 0.305
 - d. 0.402
 - e. 0.475

(1 point)

55. If a randomly selected patient has a heart attack then, what is the probability that the patient has been prescribed Drug X?
- a. 0.165
 - b. 0.305
 - c. 0.347
 - d. 0.402
 - e. 0.506

(1 point)

56. If a randomly selected patient has a heart attack then, what is the probability that the patient has been prescribed Drug Y?

- a. 0.150
- b. 0.316
- c. 0.402
- d. 0.506
- e. 0.550

(1 point)

57. If a randomly selected patient has a heart attack then, what is the probability that the patient has been prescribed Drug Z?

- a. 0.204
- b. 0.250
- c. 0.337
- d. 0.402
- e. 0.506

(1 point)

Questions 58 – 61 are based on the following information:

Sweet Cold is an ice cream supplier in a city. During the forthcoming summer, the manager of the firm expects to sell at least 8000 boxes of ice cream at a profit of Rs.15 per box. If, instead of entering the market at the beginning of the summer, he waits for some weeks then, he can increase the quantity sold by 400 additional boxes per week he waits. However, due to increase in competition, the profit margin may reduce by Re.0.50 per box per week he waits.

58. If x denotes the number of weeks that the manager waits before entering the market then, which of the following is the profit function for the firm?

- a. $f(x) = 2000 + 1,20,000x - 200x^2$
- b. $f(x) = 200 + 1,20,000x - 2000x^2$
- c. $f(x) = 1,20,000 + 2000x - 200x^2$
- d. $f(x) = 2000x - 200x^2$
- e. $f(x) = 1,20,000 + 2000x$

(1 point)

59. The number of weeks that the manager should wait before entering the market, so that the profit of the firm is maximized, is that value of x for which

- a. $y > 0$
- b. $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$
- c. $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$
- d. $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$
- e. $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} = 0$

(1 point)

60. The number of weeks that the manager should wait, so that the profit of the firm is maximized is

- a. 0
- b. 1
- c. 2
- d. 5
- e. 10

(2 points)

61. The maximum profit that can be realized by the firm is

- a. Rs. 10,000
- b. Rs. 12,500
- c. Rs. 25,000
- d. Rs.1,00,000
- e. Rs.1,25,000

(1 point)

62. The average monthly salary of the supervisors and workmen of Alpha Ltd. is Rs.6,000. The average monthly salary of the supervisors is Rs.9,000 and the average monthly salary of the workmen is Rs.5000. Moreover, the average monthly salary of the middle level managers of the company is Rs.14,000. The percentage of supervisors among the total number of workers and supervisors in the company is

- a. 10%
- b. 20%
- c. 25%
- d. 30%
- e. 40%

(3 points)

Questions 63 – 64 are based on the following information:

The number of applications received for various numbers of equity shares in the public issue of Poineer Industries Ltd. are given below:

Number of shares applied for	Number of applications
2000 – 2500	21000
2500 – 3000	18500
3000 – 3500	16000
3500 – 4000	14000
4000 – 4500	10500
4500 – 5000	3000

63. The mean number of shares applied for is

- a. 2000.4
- b. 3000.0
- c. 3150.6
- d. 3500.3
- e. 4025.5

(3 points)

64. The median number of shares applied for is

- a. 2050.3
- b. 2500.6
- c. 3000.4
- d. 3062.5
- e. 4010.5

(3 points)

Questions 65 – 72 are based on the following information:

Modern Enterprises manufactures two products, P and Q, that can be sold easily. Both the products require the raw materials X and Y respectively. Each unit of P requires total 30 kgs of raw materials X and Y in the ratio of 2:1 respectively. Each unit of Q requires 20 kgs of raw materials X and Y in the ratio of 3:2 respectively. The selling price per unit of P is Rs.150 and the profit margin on the same is 40%. The selling price per unit of Q is Rs.125 and the profit margin on the same is 32%. Presently 2500 kgs of X and 1500 kgs of Y are available in the inventory. The production manager of the company is planning the production for the next week. The policy of the company is to maintain 20% of the quantities of raw materials available at the beginning of the week, as the closing stock at the end of the week. The objective of the company is to maximize profit. A linear programming problem (LPP) has to be formulated and solved by the simplex method.

65. If P denotes number of units of the product P, Q denotes number of units of the product Q and Z denotes profit then, the objective function of the LPP is

- a. Maximize $Z = 60P + 40Q$
- b. Maximize $Z = 40P + 60Q$
- c. Maximize $Z = 20P + 12Q$
- d. Maximize $Z = 12P + 20Q$
- e. Maximize $Z = 10P + 8Q$

(1 point)

66. The constraint on the usage of raw material X is represented by

- a. $12P + 20Q \leq 2000$
- b. $20P + 12Q \leq 2000$
- c. $10P + 8Q \leq 1200$
- d. $8P + 10Q \leq 1200$
- e. $10P + 12Q \leq 2000$

(1 point)

67. The constraint on the usage of raw material Y is represented by

- a. $20P + 12Q \leq 2000$
- b. $12P + 20Q \leq 2000$
- c. $20P + 12Q \leq 1200$
- d. $8P + 10Q \leq 1200$
- e. $10P + 8Q \leq 1200$

(1 point)

68. S_1 denotes slack raw material X and S_2 denotes slack raw material Y. According to the first tableau, which of the following is/are **correct**?

- a. The variable to enter solution is P
- b. The variable to leave solution is S_1
- c. The variable to enter solution is S_1
- d. The variable to leave solution is S_2
- e. Both (a) and (b) above.

(2 points)

69. Which of the following is **correct**, according to the second tableau?

- a. The solution is to produce 100 units of P and the value of S_2 is 200
- b. The solution is to produce 200 units of P and the value of S_2 is 100
- c. The solution is to produce 200 units of P and the value of S_1 is 100
- d. The solution is to produce 100 units of Q and the value of S_1 is 200
- e. The solution is to produce 200 units of Q and the value of S_1 is 100

(2 points)

70. Which of the following is/are **correct**, according to the second tableau?

- a. The variable to enter solution is P
- b. The variable to leave solution is Q
- c. The variable to enter solution is Q
- d. The variable to leave solution is S_2
- e. Both (c) and (d) above.

(1 point)

71. Which of the following is **correct**, according to the third tableau?

- a. The solution is to produce 0 units of P and 40 units of Q
- b. The solution is to produce 40 units of P and 100 units of Q
- c. The solution is to produce 100 units of P and 0 units of Q
- d. The solution is to produce 0 units of P and 100 units of Q
- e. The solution is to produce 100 units of P and 40 units of Q.

(2 points)

72. Which of the following statements is/are **correct**?

- a. The third tableau is not optimal and the solution needs to be improved further
- b. According to the third tableau, the variable to enter solution is S_1 and the variable to leave solution is P
- c. The third tableau is optimal
- d. The maximum profit is Rs.6,400
- e. Both (c) and (d) above.

(1 point)

73. The sixth and eight terms of a harmonic progression (H.P.) are $\frac{1}{26}$ and $\frac{1}{34}$ respectively.

The first five terms of the H.P are

- a. $\frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \frac{1}{18}$ and $\frac{1}{22}$
- b. $\frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}$ and $\frac{1}{16}$
- c. $\frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}$ and $\frac{1}{13}$
- d. $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$ and $\frac{1}{12}$
- e. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$ and $\frac{1}{11}$

(2 points)

74. $\frac{\sqrt[8]{a^5} x \sqrt[16]{a^5} x \sqrt[8]{a^3} x \sqrt[16]{a^3}}{\sqrt{a}}$ is

- a. $\sqrt[4]{a}$
- b. $\sqrt[3]{a}$
- c. $\sqrt{a}$
- d. a
- e. a^2

(1 point)

75.
$$\begin{aligned} \frac{1}{2}x - \frac{1}{3}y + \frac{1}{4}z &= 12 \\ \frac{2}{10}x - \frac{1}{9}y + \frac{1}{3}z &= 8 \\ \frac{3}{10}x - \frac{2}{9}y - \frac{7}{12}z &= -2 \end{aligned}$$

The value of z which satisfies the above three simultaneous equations is

- a. 3
- b. 4
- c. 6
- d. 9
- e. 12.

(1 point)

76. If $f(x) = 3x^3 + 2x^2 - 4x + 6$, then $f'''(x)$ is

- a. -4
- b. 2
- c. 6
- d. 9
- e. 18.

(1 point)

77. If $y = x^2 - 3x + 4$, then which of the following statements is **true** for y at the point where it intersects the y -axis?

- a. y is increasing at an increasing rate
- b. y is increasing at a decreasing rate
- c. y is decreasing at an increasing rate
- d. y is decreasing at a decreasing rate
- e. None of the above.

(2 points)

78. The following data relate to the sales of National Industries Ltd.

Year	Growth in sales (%)
1999	25
2000	30
2001	06

The compounded annual growth rate of sales is approximately

- a. 6.67%
- b. 12.84%
- c. 15.25%
- d. 19.87%
- e. 28.80%.

(1 point)

79. If 5 is subtracted from every item in a data set then the coefficient of variation of the resulting data set is 10%. If 5 is added to every item of the same data set then the coefficient of variation of the resulting data set is 6%.

The coefficient of variation of the original data set is

- a. 6.0%
- b. 7.5%
- c. 10.0%
- d. 12.5%
- e. 15.6%.

(3 points)

80. The roots of the equation: $6x^2 + 23x + 20 = 0$ are

- a. $-\frac{4}{3}$ and $\frac{5}{2}$
- b. $\frac{4}{3}$ and $-\frac{5}{2}$
- c. $-\frac{4}{3}$ and $-\frac{5}{2}$
- d. $\frac{4}{3}$ and $\frac{5}{2}$
- e. 2 and $\frac{5}{3}$.

(1 point)

81. There are three numbers, A, B and C. The sum of two times the logarithm of A with respect to base 2, three times the logarithm of B with respect to base 3, and four times the logarithm of C with respect to base 4, is equal to 18. The sum of four times the logarithm of A with respect to base 2, five times the logarithm of B with respect to base 3, and six times the logarithm of C with respect to base 4, is equal to 30. The sum of eight times the logarithm of A with respect to base 2, seven times the logarithm of B with respect to base 3, and five times the logarithm of C with respect to base 4 is equal to 40.

What is the sum of the numbers A, B and C?

- a. 4
- b. 6
- c. 16
- d. 29
- e. 58.

(4 points)

82. The sum of three consecutive numbers of an arithmetic progression (A.P.) is 12. If the third number is increased by 2 then, the three consecutive numbers form a geometrical progression (G.P.).

Which of the following statements is/are **correct**?

- I. The second term of the A.P. is 0
 - II. The first term of the A.P. is 0 or 4
 - III. The first term of the A.P. is 6 or 8
 - IV. The first term of the A.P. is 2 or 8
 - V. The third term of the A.P. is 4 or 8
 - VI. The third term of the A.P. is 6 or 4
 - VII. The third term of the A.P. is 6 or 0
 - VIII. The second term of the A.P. and G.P. are different
- a. Both (I) and (II) above
 - b. Both (III) and (V) above
 - c. Both (II) and (VI) above
 - d. Both (IV) and (VII) above
 - e. Both (VII) and (VIII) above.

(2 points)

END OF SECTION B

Suggested Answers

Quantitative Methods – I (131) : April 2003

Section A : Basic Concepts

1. Answer : (c)
Reason : The set of natural numbers, N, includes all the numbers starting from 1 and obtained by adding 1 to the previous numbers. N does not include 0 and the negative numbers. Further N includes both even and odd numbers
2. Answer : (e)
Reason : The set of real numbers excludes the complex numbers. It includes the natural numbers, integers, rational numbers and whole numbers.
3. Answer : (a)
Reason : $(a + b) + c = a + (b + c)$ is known as the associative property of addition.
4. Answer : (d)
Reason : The inverse property of multiplication states that for every number a, ($a \neq 0$), there exists another number $\frac{1}{a}$, such that the multiplication of a and $\frac{1}{a}$ is equal to 1.
5. Answer : (a)
Reason : (a, b) implies the set of all real numbers, x, such that, $a < x < b$. Therefore (a, b) excludes all real numbers $x \leq a$ and $x \geq b$.
 $[a, b]$ includes all real numbers, x, such that $a \leq x \leq b$.
 $(a, b]$ includes all real numbers, x, such that $a < x \leq b$
 $[a, b)$ includes all real numbers, x, such that $a \leq x < b$.
6. Answer : (c)
Reason : The least common multiple of a group of quantities can be divided by each quantity in the group without leaving any remainder. While, each number of the group can be divided by a factor, even by the highest common factor also. But that condition is not satisfied, if each number is divided by the average or the sum of all the quantities.
7. Answer : (c)
Reason : The roots of the equation $ax^2 + bx + c = 0$ are
$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2 \cdot a}$$

Now, the above equation will have a single root if and only if $b^2 - 4ac = 0$ or, $\frac{b^2}{4} = ac$.
8. Answer : (e)
Reason : A series of numbers is said to be in AP, if the consecutive numbers in the series increase or decrease by a constant value, and neither by a fixed factor nor by a fixed multiple. Moreover, the consecutive numbers need not necessarily experience any change in sign, nor be in multiples of 100.
9. Answer : (c)
Reason : In any arithmetic progression, the n-th term can be expressed as: $t_n = a + (n - 1) d$
Where a is the first number and d is the common difference.

10. Answer : (e)

Reason : The sum of n terms in an AP is

$$S_n = \frac{n}{2} \{2a + (n-1)d\}$$

Now, if $S_n = 0$ then $\frac{n}{2} \{2a + (n-1)d\} = 0$ (where $n \neq 0$)

$$\text{So, } 2a + (n-1)d = 0 \quad \text{or, } a = -\frac{(n-1)d}{2}$$

11. Answer : (d)

Reason : If a, b and c are in G.P., then $\frac{a}{b} = \frac{b}{c}$

$$\text{or, } b^2 = ac$$

12. Answer : (e)

Reason : A harmonic progression is formed by taking the reciprocals of the corresponding arithmetic progression, and not in geometric progression. These numbers need not necessarily be odd numbers or in multiples of 3 or even numbers in multiples of 4.

13. Answer : (b)

Reason : If a, b and c are in harmonic progression, then $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ or, $\frac{2}{b} = \frac{a+c}{ac}$

$$\text{or, } 2ac = ab + bc \quad \text{or, } ac - bc = ab - ac$$

$$\text{or, } ab - ac = ac - bc$$

$$\text{or, } a(b-c) = c(a-b)$$

14. Answer : (e)

Reason : Here, $A = \frac{a+b}{2}$, $G = \sqrt{a \cdot b}$ i.e. $G^2 = ab$

$$\text{and } H = \frac{2ab}{a+b}$$

From the above three relationship, $H = \frac{G^2}{A}$ or, $G^2 = A.H$

15. Answer : (d)

$$\text{Reason : } n_{C_r} = \frac{n!}{n-r!r!} \quad n_{P_r} = \frac{n!}{n-r!}$$

$$\therefore n_{C_r} = n_{P_r} \div r!$$

16. Answer : (d)

Reason : If $a = \log_b c$, then from the principle of logarithm, $c = b^a$

17. Answer : (b)

Reason : According to the principle of logarithm,

$$\log_a \left(\frac{M}{N} \right) = \log_a (M) - \log_a (N)$$

18. Answer : (a)
Reason : If, $\log_a 1 = x$, then $a^x = 1$
The above relationship is only possible, if $x = 0$
So, $\log_a 1 = 0$
19. Answer : (d)
Reason : The absolute value of any number, say a , is given by
 $|a| = a$, if $a < 0$ and $|a| = a$, if $a > 0$.
i.e. the magnitude of the number is only considered, not the sign.
20. Answer : (e)
Reason : For any real number, a , the interval $[a, +\infty)$ indicates the set of all real numbers greater than or equal to a , it includes all the even and odd numbers greater than or equal to a .
21. Answer : (b)
Reason : The range of a function, $y = f(x)$ includes, all the values of y which can be associated with the values of x .
22. Answer : (a)
Reason : For a function $y = f(x)$, every member in the domain is associated with only one member in the range.
23. Answer : (e)
Reason : The limiting value of a function, $y = f(x)$ at $x = a$ is defined as the value to which y approaches, as x approaches a ; it is neither the minimum value, nor the maximum value.
24. Answer : (e)
Reason : The derivative of a function, $y = f(x)$ at $x = a$ is equal to
$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$
. It indicates the rate at which $f(x)$ is changing with respect to the corresponding change in x .
25. Answer : (e)
Reason : If, for any function $y = f(x)$, $f'(x) > 0$ and $f''(x) < 0$ then the function is said to be increasing at a decreasing rate.
26. Answer : (a)
Reason : For any function $y = f(x)$, if $f'(x) = 0$ at $x = a$, the function is said to have a relative maxima or minima.
27. Answer : (d)
Reason : Linear programming assumes that the decision variables can assume any non-negative value and not only integral values.
28. Answer : (c)
Reason : In the graphical method of solving LPP, the points lying outside the feasible region do not satisfy the constraints. All other statements are true with regard to the LPP.
29. Answer : (a)
Reason : The optimal solution to a LPP by the graphical method occurs at the corner points of the feasible region. It does not occur either at the center or outside the feasible region. It need not necessarily be at the horizontal or vertical axis.
30. Answer : (c)
Reason : The variable to enter solution, in the simplex method, is indicated by the $Z_j - C_j$ values and not by the solution column. All other statements are true with regard to the simplex method.

31. Answer : (b)

Reason : A positively skewed distribution is a right tailed one where $AM > Median > Mode$ and the reverse is true for a negatively skewed distribution. But for a symmetrical distribution, all the arithmetic mean, median and mode are equal. Only for a negatively skewed distribution, most of the values are concentrated at the higher end of the distribution.

32. Answer : (d)

Reason : A frequency distribution is represented by a histogram, using rectangles where the width and height of the rectangles are proportionate to the respective class intervals and class frequencies. While plotting bi-variate data in two-dimensions, scatter diagram is obtained. Frequency polygon is prepared by joining the mid-points at the top of the rectangles in a histogram by lines. In an ogive, a cumulative frequency distribution (less than or more than) is presented.

33. Answer : (c)

Reason : Let there be n-number of observations $x_1, x_2, \dots, x_n$ and their arithmetic mean

$$\bar{x} = \frac{1}{n} \{x_1 + x_2 + x_3 + \dots + x_n\}$$

Now, if all the items are multiplied by k then the arithmetic mean

$$\bar{y} = \frac{1}{n} \{kx_1 + kx_2 + \dots + kx_n\} = k \cdot \frac{1}{n} \{x_1 + x_2 + \dots + x_n\} = k \cdot \bar{x}$$

Therefore, if all the items in a data set is multiplied by k, the mean of the resulting data set will be k times the mean of the original data set.

34. Answer : (e)

Reason : Arithmetic, geometric and harmonic means are calculated by mathematical treatment. The median is a positional average. But mode is the most frequently occurring value in a set of observations.

35. Answer : (b)

Reason : Geometric mean is the appropriate measure for computing the average rate of change of quantities that change over time. Other types of averages are not appropriate.

36. Answer : (a)

Reason : Simple arithmetic mean is the appropriate mean for a set of ratios which have been calculated with the same denominators. Other types of average are not appropriate.

37. Answer : (a)

Reason : According to the Bienayme-Chebyshev rule, the percentage of data points lying within $\pm k$ standard deviation of the mean is at least $\left(1 - \frac{1}{k^2}\right) \times 100$.

If $k = 1$, then it will be 0 percent which is impossible. So this rule is not applicable for $k = 1$

38. Answer : (c)

Reason : By the application of Baye's theorem, posterior or revised probability is calculated.

39. Answer : (c)

Reason : Co-efficient of variation, a relative measures of dispersion, is used to compare the variability, homogeneity, stability, uniformity and consistency of two or more series. While variance, standard deviation and mean deviation are the absolute measures of dispersion and can not be used for comparison.

40. Answer : (e)

Reason : If two events A and B are mutually exclusive, then

$$P(AB) = 0 \text{ or } P(A).P(B/A) = 0$$

$$\text{Or, } P(B).P(A/B) = 0$$

$$\text{But } P(A) \neq 0 \text{ and } P(B) \neq 0$$

$$\text{Hence, } P(A/B) = P(B/A) = 0$$

Section B : Problems

41. Answer : (d)

Reason : If A, G and H are the arithmetic, geometric and harmonic means between two quantities then their relationship is: $G^2 = AH$

$$\text{Given: } A = 4 \quad G = 6$$

$$\therefore 6^2 = 4H$$

$$\text{or } H = \frac{6^2}{4} = 9$$

42. Answer : (b)

$$\text{Reason : } \log_2 9 \times \log_3 2 = \log_2 3^2 \times \log_3 2 = 2 \log_2 3 \times \log_3 2 = 2(\log_2 3 \times \log_3 2) = 2 \times 1 = 2$$

43. Answer : (e)

Reason : The third term is the geometric mean of the second and fourth term.

$$\therefore \text{Third term} = (10 \times 40)^{\frac{1}{2}} = 20$$

$$\frac{\text{Fourth term}}{\text{Third term}} = r \quad (\text{Common ratio}) \quad \therefore r = \frac{40}{20} = 2$$

$$\text{Fifth term} = \text{Fourth term} \times r = 40 \times 2 = 80.$$

44. Answer : (b)

$$\text{Reason : Number of permutations of five things taking 2 things at a time} = {}^5P_2 = 20$$

45. Answer : (a)

$$\text{Reason : If } f(x) = e^{g(x)} \text{ then, } f'(x) = g'(x).e^{g(x)}$$

$$\text{Given : } f(x) = e^{x^2+5}$$

$$\therefore f'(x)$$

46. Answer : (b)

$$\text{Reason : Standard deviation} = (\text{variance})^{\frac{1}{2}} = (9)^{\frac{1}{2}} = 3$$

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

47. Answer : (c)

Reason : For non-mutually exclusive and independent events A and B,

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$P(A) = 0.2$$

$$P(B) = 0.3$$

$$P(A \text{ and } B) = 0.2 \times 0.3 = 0.06$$

48. Answer : (e)

Reason : For a given data set, if each observation is multiplied by a constant then, the standard deviation of the resulting data set is equal to the product of the standard deviation of the original data set and the constant. In this case standard deviation of the original data set = $\sqrt{3}$

$$\therefore \text{Standard deviation of the resulting data set} = 2\sqrt{3}$$

$$\therefore \text{Variance of the resulting data set} = (2\sqrt{3})^2 = 4 \times 3 = 12.$$

49. Answer : (a)

$$\begin{aligned} \text{Reason : } \lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x - 3} &= \lim_{x \rightarrow 3} \frac{x^2 - 3x - 2x + 6}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{x(x - 3) - 2(x - 3)}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{(x - 2)(x - 3)}{(x - 3)} \quad (\text{for } x \neq 3) \end{aligned}$$

50. Answer : (b)

$$\begin{aligned} \text{Reason : } P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ \text{or } P(B) &= P(A \text{ or } B) - P(A) + P(A \text{ and } B) \\ &= 0.80 - 0.50 + 0.30 \\ &= 0.60 \end{aligned}$$

Answers 51 through 57:

Let us denote the events as follows:

$$X: \text{ A randomly selected patient has been prescribed Drug X. } P(X) = \frac{30}{30 + 30 + 40} = 0.30$$

$$Y: \text{ A randomly selected patient has been prescribed to take Drug Y. } P(Y) = \frac{30}{30 + 30 + 40} = 0.30$$

$$Z: \text{ A randomly selected patient has been prescribed to take Drug Z. } P(Z) = \frac{40}{30 + 30 + 40} = 0.40$$

H: Occurrence of heart attack.

So $P(X) = 0.3$, $P(Y) = 0.3$ and $P(Z) = 0.4$

$P(H/X)$: Probability that a patient will suffer heart attack, provided he has been prescribed drug X

$$P(H/X) = 0.75 - 0.20 = 0.55,$$

Similarly $P(H/Y) = 0.75 - 0.25 = 0.50$ and

$$P(H/Z) = 0.75 - 0.35 = 0.40$$

51. Answer : (a)

Reason : $P(X \text{ and } H)$: Probability that a patient has been prescribed drug X and he will suffer heart attack.

$$P(X \text{ and } H) = P(X) \cdot P(H/X)$$

$$P(X \text{ and } H) = 0.55 \times 0.3 = 0.165$$

52. Answer : (a)

$$\text{Reason : } P(Y \text{ and } H) = P(Y) \cdot P(H/Y)$$

$$P(Y \text{ and } H) = 0.5 \times 0.3 = 0.150$$

53. Answer : (b)

$$\text{Reason : } P(Z \text{ and } H) = P(Z) \cdot P(H/Z)$$

$$P(Z \text{ and } H) = 0.4 \times 0.4 = 0.160$$

54. Answer : (e)

Reason : Hence, the probability of a heart attack when either of the three drugs has been prescribed,

$$\begin{aligned} P(H) &= P(X \text{ and } H) + P(Y \text{ and } H) + P(Z \text{ and } H) \\ &= 0.165 + 0.150 + 0.160 = 0.475 \end{aligned}$$

55. Answer : (c)

Reason : The probability that a patient who has suffered heart attack has been prescribed Drug X

$$= P(X/Y) = \frac{P(X \text{ and } H)}{P(H)} = \frac{0.165}{0.475} = 0.347$$

56. Answer : (b)

Reason : The probability that a patient who has suffered heart attack has been prescribed Drug Y

$$= P(Y/H) = \frac{P(Y \text{ and } H)}{P(H)} = \frac{0.150}{0.475} = 0.316$$

57. Answer : (c)

Reason : The probability that a patient who has suffered heart attack has been prescribed Drug Z

$$= P(Z/H) = \frac{0.160}{0.475} = 0.337$$

58. Answer : (c)

Reason : The amount of profit per box is Rs.15

Let, the waiting period be x.

Therefore, the number of boxes that may be sold = $8,000 + 400x$

And, total profit (y) will be

$$\begin{aligned} y &= (8000 + 400x)(15 - 0.5x) \\ &= 1,20,000 + 6,000x - 4,000x - 200x^2 \\ &= 1,20,000 + 2,000x - 200x^2 \end{aligned}$$

59. Answer : (b)

Reason : For the maximum value of y, $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

60. Answer : (d)

Reason : By differentiating y with respect to x,

$$\frac{dy}{dx} = 2000 - 400x$$

$$\text{and } \frac{d^2y}{dx^2} = -400$$

$$2000 - 400x = 0$$

$$\text{or } x = \frac{2000}{400} = 5$$

Therefore, the sales manager should wait for 5 weeks to maximize the profit.

61. Answer : (e)

Reason : The maximum amount of profit will be

$$Y_{\max} = (8000 + 400 \times 5) (15 - 0.5 \times 5)$$

$$= 10,000 \times 12.50 = \text{Rs.1,25,000}$$

62. Answer : (c)

Reason : Let N_1 denote the number of supervisors and N denote the total number of supervisors and workmen

$$\bar{X}_{12} = \frac{N_1 \bar{X}_1 + (N - N_1) \bar{X}_2}{N}$$

$$\text{or } \frac{9000 N_1 + 5000 (N - N_1)}{N} = 6000$$

$$\text{or } \frac{9000 N_1 + 5000 N - 5000 N_1}{N} = 6000$$

$$\text{or } \frac{4000 N_1 + 5000 N}{N} = 6000$$

$$\text{or } \frac{4000 N_1}{N} + 5000 = 6000$$

$$\text{or } \frac{4000 N_1}{N} = 1000$$

$$\text{or } \frac{N_1}{N} = \frac{1000}{4000} = \frac{1}{4} = 0.25$$

$\therefore$ Percentage of supervisors = $0.25 \times 100 = 25\%$.

63. Answer : (c)

Reason :

No. of shares (1)	Mid value(x) (2)	Frequency ('000) (f) (3)	(2) $\times$ (3) ('000)
2000 - 2500	2250	21	47250
2500 - 3000	2750	18.50	50,875
3000 - 3500	3250	16	52,000
3500 - 4000	3750	14	52,500
4000 - 4500	4250	10.50	44,625
4500 - 5000	4750	3	14,250
		$\Sigma f = 83$	$\Sigma fx = 261500$

$$\text{Mean number of applications} = \frac{\Sigma fx}{\Sigma f} = \frac{261500 \times 1000}{83 \times 1000} = 3150.6$$

64. Answer : (d)

Reason :

No of shares	Frequency ('000)	Cumulative Frequency ('000)
2000 - 2500	21	21
2500 - 3000	18.50	39.50
3000 - 3500	16.00	55.50
3500 - 4000	14.00	69.50
4000 - 4500	10.50	80.00
4500 - 5000	3.00	83.00

$$\text{Median position} = \frac{N+1}{2} = \frac{83000+1}{2} = 41500.5$$

∴ Median class is 3000 – 3500.

$$\therefore \text{Median} = L_M + \left[\frac{\left(\frac{(N+1)}{2} - (F+1) \right)}{f_m} \right] W$$

$$\therefore \text{Median number of applications} = 3000 + \left[\frac{41500.5 - 39501}{16000} \right] \times 500 = 3062.5$$

Answers 65 through 72:

Let, the number of units of products P and Q be denoted by P and Q units respectively. $P, Q \geq 0$

65. Answer : (a)

Reason : Selling price per unit of P = Rs.150

Profit margin = 40%

$$\text{Profit per unit} = \frac{150 \times 40}{100} = \text{Rs.60}$$

Selling price per unit of Q = Rs.125

Profit margin = 32%

$$\text{Profit per unit} = \frac{125 \times 32}{100} = \text{Rs.40}$$

∴ Total profit, $Z = 60P + 40Q$

∴ Objective function of the LPP is

Maximize $Z = 60P + 40Q$

66. Answer : (b)

Reason : The company maintains 20% of the quantities of raw materials available at the beginning of the week, as closing stock at the end of the week.

So, usable quantity of raw material X = 80% of 2500 kg = 2000 kg

Usable quantity of raw material Y = 80% of 1500 kg = 1200 kg.

Each unit of P requires:

$$\frac{2}{3} \times 30 = 20 \text{ kgs of X,}$$

$$\text{and } \frac{1}{3} \times 30 = 10 \text{ kgs of Y,}$$

Each unit of Q requires:

$$\frac{3}{5} \times 20 = 12 \text{ kgs of X,}$$

$$\text{and } \frac{2}{5} \times 20 = 8 \text{ kgs Y,}$$

So, constraint on the raw-material X is,

$$20P + 12Q \leq 2000 \dots\dots(1)$$

67. Answer : (e)

Reason : Constraint on the raw material Y is $10P + 8Q \leq 1200 \dots\dots (2)$

68. Answer : (e)

Reason : The above inequations (1) and (2) can be rewritten as

$$20P + 12Q + S_1 = 2000 \dots\dots\dots(3)$$

$$10P + 8Q + S_2 = 1200 \dots\dots\dots(4) \text{ where } S_1, S_2 \geq 0$$

(S_1 and S_2 are the slack variables)

The objective function can be rewritten as: maximize: $60p + 40Q + 0.S_1 + 0.S_2$

Tableau – 1

			60	40	0	0	Ratio
Profit	Variable	Solution	P	Q	S_1	S_2	
0	S_1	2000	(20)	12	1	0	100
0	S_2	1200	10	8	0	1	120
$Z_j - C_j$		0	-60	-40	0	0	

$$\text{Enter: P} \quad \text{Leave: Min} \left(\frac{2000}{20}, \frac{1200}{10} \right) = \text{Min. (100, 120) : } S_1$$

$\therefore$ P will enter the solution and S_1 will leave solution.

69. Answer : (a)

Reason : **Tableau-2**

			60	40	0	0	Ratio
Profit	Variable	Solution	P	Q	S_1	S_2	
60	P	100	1	0.6	0.05	0	$\frac{100}{0.6} = 166.67$
0	S_2	200	0	(2)	-0.5	1	$\frac{200}{2} = 100$
$Z_j - C_j$		6000	0	-4	3	0	

Enter :Q Leave : S_2

$\therefore$ The solution is to produce 100 units of P and the value of S_2 is 200.

70. Answer : (e)

Reason : Q will enter solution and S_2 will leave solution, according to the second tableau.

71. Answer : (b)

Reason : **Tableau-3**

			60	40	0	0
Profit	Variable	Solution	P	Q	S_1	S_2
60	P	40	1	0	0.20	-0.3
40	Q	100	0	1	-0.25	0.5
$Z_j - C_j$		6,400	0	0	2	2

$\therefore$ 40 units of P and 100 units of Q should be produced according to the third tableau.

72. Answer : (e)

Reason : The solution according to the third tableau is optimal because all the $Z_j - C_j$ values are non-negative. Moreover the maximum profit is Rs.6,400.

73. Answer : (a)

Reason : The 6th and 8th term of the HP are $\frac{1}{26}$ and $\frac{1}{34}$ respectively. Therefore, the 6th and 8th term of the corresponding AP are 26 and 34 respectively.

Let the first term in the AP be a and the common difference be d.

So, the sixth term = $a + 5d = 26$ (1)

and the eight term = $a + 7d = 34$ (2)

Subtracting the equation (1) from the equation (2).

$$\begin{array}{rcl} a + 7d & = & 34 \\ a + 5d & = & 26 \\ \hline & & 2d = 8 \end{array} \quad \text{or, } d = 4$$

Putting the value of d = 4 in the equation (1):

$$a + 5 \times 4 = 26$$

$$\text{or } a = 26 - 20 = 6$$

∴ The AP is: 6, 10, 14, 18, 22.

∴ The corresponding HP is: $\frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \frac{1}{18}$ and $\frac{1}{22}$

74. Answer : (d)

Reason : Simplifying the given expression we get

$$a \left\{ \left(\frac{5}{8} + \frac{5}{16} + \frac{3}{8} + \frac{3}{16} \right) - \frac{1}{2} \right\} = a \frac{8}{8} = a$$

75. Answer : (e)

Reason : By subtracting the second equation and the third equation from the first equation we get

$$\left(\frac{1}{2} - \frac{2}{10} - \frac{3}{10} \right)x - \left(\frac{1}{3} - \frac{1}{9} - \frac{2}{9} \right)y + \left(\frac{1}{4} - \frac{1}{3} + \frac{7}{12} \right)z = 12 - 8 - (-2)$$

$$\text{or } 0.x - 0.y + \frac{3-4+7}{12}z = 6$$

$$\text{or } z = \frac{6}{6} \times 12 = 12$$

76. Answer : (e)

$$\text{Reason : } \frac{df}{dx} = 9x^2 + 4x - 4; \frac{d^2f}{dx^2} = 18x + 4; \frac{d^3f}{dx^3} = 18$$

77. Answer : (c)

Reason : At y-axis the x – co-ordinate is always zero. Here $\frac{dy}{dx} = 2x - 3 = -3$ and $\frac{d^2y}{dx^2} = 2$; For x = 0,

$\frac{dy}{dx} = -3$ (a negative value) and $\frac{d^2y}{dx^2} = 2$ (a positive value). So, we can say that y is decreasing at an increasing rate.

78. Answer : (d)

$$\begin{aligned} \text{Reason : Compounded annual growth rate of sales} &= [1.25 \times 1.30 \times 1.06]^{1/3} - 1 = (1.7225)^{1/3} - 1 \\ &= 1.1987 - 1 = 0.1987 \text{ i.e., } 19.87\% \end{aligned}$$

79. Answer : (b)

Reason : Let the average and standard deviation of the original data let be $\bar{x}$ and s .

$$\text{Average of all items 'x - 5'} = \frac{\Sigma(x-5)}{n} = \frac{\Sigma x}{n} - \frac{\Sigma 5}{n} = \bar{x} - \frac{n5}{n} = \bar{x} - 5.$$

Standard deviation of all items 'x - 5' = s

(This is because the value of standard deviation remains the same if each observation in a series is increased or decreased by the same quantity).

$$\text{Given: } \frac{s}{\bar{x} - 5} \times 100 = 10$$

$$\text{or } 100s = 10\bar{x} - 50$$

$$\text{or } 10\bar{x} - 100s = 50 \dots\dots\dots (A)$$

$$\text{Average of all items 'x + 5'} = \frac{\Sigma(x+5)}{n} = \frac{\Sigma x}{n} + \frac{\Sigma 5}{n} = \bar{x} + \frac{n5}{n} = \bar{x} + 5$$

Standard deviation of all items 'x + 5' = s

(This is because the value of standard deviation remains the same if each observation in a series is increased or decreased by the same quantity).

$$\text{Given : } \frac{s}{\bar{x} + 5} \times 100 = 6$$

$$\text{or } 100s = 6\bar{x} + 30$$

$$\text{or } 6\bar{x} - 100s = -30 \dots\dots\dots (B)$$

Subtracting equation (B) from equation (A) we get:

$$(10\bar{x} - 100s) - (6\bar{x} - 100s) = 50 - (-30)$$

$$\text{or } 4\bar{x} = 80$$

$$\text{or } \bar{x} = \frac{80}{4} = 20$$

$$\therefore \bar{x} = 20.$$

Putting the value of $\bar{x}$ in equation (A) we get :

$$10(20) - 100s = 50$$

$$\text{or } 200 - 100s = 50$$

$$\text{or } 100s = 150$$

$$\text{or } s = \frac{150}{100} = 1.50$$

$$\therefore s = 1.50.$$

$$\text{Coefficient of variation of the original data set} = \frac{s}{\bar{x}} \times 100$$

$$= \frac{1.50}{20} \times 100$$

80. Answer : (c)

$$\text{Reason : } 6x^2 + 23x + 20 = 0$$

$$a = 6; \quad b = 23; \quad c = 20$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-23 \pm \sqrt{(23)^2 - (4 \times 6 \times 20)}}{2 \times 6} = \frac{-23 \pm \sqrt{49}}{12} = \frac{-23 \pm 7}{12}$$

$$\therefore x = \frac{-23+7}{12} \text{ or } \frac{-23-7}{12} \text{ i.e. } x = \frac{-4}{3} \text{ or } \frac{-5}{2}$$

81. Answer : (d)

Reason : Given: $2\log_2 A + 3\log_3 B + 4\log_4 C = 18 \dots (1)$

$$4\log_2 A + 5\log_3 B + 6\log_4 C = 30 \dots (2)$$

$$6\log_2 A + 4\log_3 B + 10\log_4 C = 40 \dots (3)$$

Multiplying equation (1) by 2 and subtracting equation (2) from the modified equation (1) we get

$$\begin{array}{rclcl} 4\log_2 A & + & 6\log_3 B & + & 8\log_4 C & = & 36 \\ 4\log_2 A & + & 5\log_3 B & + & 6\log_4 C & = & 30 \\ \hline & & \log_3 B & + & 2\log_4 C & = & 6 \end{array} \dots (4)$$

Multiplying equation (2) by 3 and equation (3) by 2, and subtracting the modified equation (3) from the modified equation (2) we get

$$\begin{array}{rclcl} 12\log_2 A & + & 15\log_3 B & + & 18\log_4 C & = & 90 \\ 12\log_2 A & + & 8\log_3 B & + & 20\log_4 C & = & 80 \\ \hline & & 7\log_3 B & - & 2\log_4 C & = & 10 \end{array} \dots (5)$$

Adding equations (4) and (5) we get

$$\begin{array}{rclcl} \log_3 B & + & 2\log_4 C & = & 6 \\ 7\log_3 B & - & 2\log_4 C & = & 10 \\ \hline 8\log_3 B & & & = & 16 \\ \hline \therefore \log_3 B & & & = & 2 \end{array}$$

Substituting the value of $\log_3 B$ in equation (4) we get

$$\begin{array}{rclcl} 2 & + & 2\log_4 C & = & 6 \\ \text{or} & & 2\log_4 C & = & 4 \\ \therefore & & \log_4 C & = & 2 \end{array}$$

Substituting the values of $\log_3 B$ and $\log_4 C$ in equation (1) we get

$$\begin{array}{rclcl} 2\log_2 A & + & 3 \times 2 & + & 4 \times 2 & = & 18 \\ \text{or} & 2\log_2 A & + & 6 & + & 8 & = & 18 \\ \text{or} & 2\log_2 A & = & 18 - 14 = 4 \\ \therefore & \log_2 A & = & 2 \end{array}$$

From above we get the following:

$$\begin{array}{lll} \log_2 A = 2 & \log_3 B = 2 & \log_4 C = 2 \\ \therefore A = 2^2 & \therefore B = 3^2 & \therefore C = 4^2 \\ \text{or } A = 4 & \text{or } B = 9 & \text{or } C = 16 \\ \therefore A + B + C = 29. \end{array}$$

82. Answer : (d)

Reason : Let the terms be:

$$\begin{array}{rclcl} a-d, a, a+d & & & & \\ \therefore a-d + a + a+d & = & 12 & & \\ \text{or } 3a & = & 12 & & \\ a & = & \frac{12}{3} = 4 & & \end{array}$$

Also, $(a-d)$, a , $(a+d+2)$ are in G.P.

$$\therefore \frac{a}{a-d} = \frac{a+d+2}{a}$$

$$\text{or } a^2 = (a-d)(a+d+2)$$

$$\text{or } 16 = (4-d)(6+d)$$

$$\text{or } 16 = 24 + 4d + -6d - d^2$$

$$\text{or } 16 = 24 - 2d - d^2$$

$$\text{or } d^2 + 2d + 16 - 24 = 0$$

$$\text{or } d^2 + 2d - 8 = 0$$

$$\text{or } d^2 + (4-2)d - 8 = 0$$

$$\text{or } d^2 + 4d - 2d - 8 = 0$$

$$\text{or } d(d+4) - 2(d+4) = 0$$

$$\text{or } (d-2)(d+4) = 0$$

$$\therefore d=2 \text{ or } d=-4$$

The first, second and third terms of the A.P. are

$$\begin{array}{lll} \text{for } a=4, d=2 & t_1 = a-d = 4-2 & =2 \\ & t_2 = a & = 4 \\ & t_3 = a+d = 4+2 & = 6 \end{array}$$

$$\begin{array}{lll} \text{for } a=4, d=-4 & t_1 = a-d = 4-(-4) & = 8 \\ & t_2 = a & = 4 \\ & t_3 = a+d = 4+(-4) & = 0 \end{array}$$

$\therefore$ First term of the A.P. is 2 or 8, and third term is 6 or 0. The second term of the A.P. and G.P. are same. Hence statements IV and VII are correct.